Comparison of Topology Optimization Methods for Cogging Torque Reduction of Permanent Magnet BLDC Motor

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Abstract —In this paper, the comparison between continuum and discrete design sensitivity analysis of the topology optimization for electromagnetic devices is introduced. Based on the stator pole optimization of permanent magnet brushless DC (BLDC) motor, the numerical analysis of discrete design sensitivity analysis coupled with the level set used to describe the optimization process of the stator pole is derived. The results and efficiency of stator pole shape optimization by using both continuum design sensitivity analysis and discrete design sensitivity analysis are compared, and the comparison suggests that the continuum design sensitivity analysis features high computation speed and realizability.

I. INTRODUCTION

In the case of the electromagnetic devices, the inverse problem is always one of the main research directions in the terms of electromagnetic engineering application. As the development of the computer technology, as well as the electromagnetic numerical calculation theory and analysis method, many solutions have been developed. In the recent years, there are some methods applied widely, such as response surface methodology, random optimization algorithm, certainty optimization method based on sensitivity analysis and so on [1].

II. SENSITIVITY ANALYSIS METHOD

As the sensitivity analysis, the first or second-order derivatives of the objective function and constraint need to be calculated when an optimization method is in iteration. The continuum design sensitivity analysis and discrete design sensitivity analysis are two frequently methods [2, 3]. The sensitivity calculation by using discrete design sensitivity analysis based to a typical given problem. However, the sensitivity may be deduced from the variation equations of the system in continuum design sensitivity analysis, which possesses coding is prone to coding and high optimization efficiency. The detailed analysis will be given in the full paper.

III. BOUNDARY TOPOLOGY OPTIMIZATION BASED ON LEVEL SET

The stator role shape of the permanent magnet BLDC motor is optimized to reduce the cogging torque. The boundary of the level set is used to describe the change of the stator role shape efficiently [4] . The boundary variation may be described as

$$\begin{cases} \phi(x,t) > 0 & \text{in } \Omega(t) \\ \phi(x,t) = 0 & \text{on } \partial\Omega \\ \phi(x,t) < 0 & \text{in } R^n \setminus \bar{\Omega}(t) \end{cases} \quad (1)$$

where, the zero level set $\phi(x,t)=0$ describes the material boundaries of the domain $\Omega(t)$. In the case of motor, the domain $\Omega(t)$ may be to describe the stator and the $\phi(x,t)=0$, the zero level set, distinguishes the material boundaries between the stator poles and the airgap. Fig. 1 shows the conceptual description for the level set.

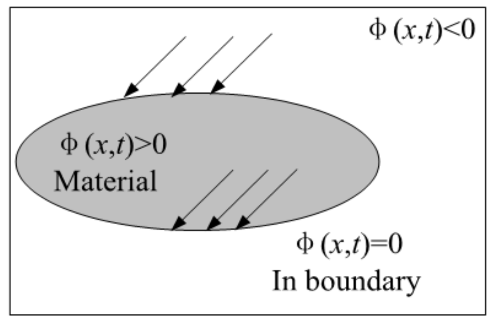

Fig. 1. Boundary representation using level set

The boundary may be adjusted by solving

$$\frac{d\phi(x(t),t)}{dt} = \frac{\partial\phi(x(t),t)}{\partial t} + \frac{\partial\phi}{\partial x}\frac{dx}{dt} = 0 \quad (2)$$

The detailed theory of Level set will be presented in full paper.

IV. NUMERICAL COMPUTATION OF DISCRETE DESIGN SENSITIVITY

The cogging torque of permanent magnet BLDC motor after Finite element discretization which is calculated by using Maxwell Stress Tensor is written as

$$T = \frac{L}{g}\sum_{i=1}^{N_{airgap}} \frac{\boldsymbol{B}_{ri}\boldsymbol{B}_{\theta i}}{\mu_0} A_i R_i H(\phi_i) \quad (3)$$

Where A_i is the area of the ith of element in airgap, R_i is the distance between the centers of the ith of element and shaft, N_{airgap} is the number of meshing elements in the airgap, $H(\phi_i)$ is the sign distance function which is related to the boundary of the level set.

The equation of design sensitivity analysis based on adjoint variable is described as

$$\frac{dF}{d\boldsymbol{\alpha}} = \frac{\partial f}{\partial \boldsymbol{\alpha}} + \frac{\partial}{\partial \boldsymbol{\alpha}}\left[\tilde{\lambda}^{\mathrm{T}}\boldsymbol{J}(\boldsymbol{\alpha}) - \tilde{\lambda}^{\mathrm{T}}\boldsymbol{K}(\boldsymbol{\alpha})\tilde{\Phi}\right] \quad (4)$$

Where, $\tilde{\lambda}$ is the adjoint variable. $\boldsymbol{J}(\boldsymbol{\alpha})$ is current density and $\boldsymbol{K}(\boldsymbol{\alpha})$ is the stiffness matrix of finite element. $\boldsymbol{\alpha}$ is optimal design variables, such as reluctivity in this paper.

The partial derivative of (3) with respect to magnetic vector potential **A** may be calculated as

$$\frac{\partial T}{\partial \boldsymbol{A}} = \frac{L}{g}\sum_{i=1}^{N_{airgap}} \frac{R_i A_i}{\mu_0}\left(\frac{\partial \boldsymbol{B}_{ri}}{\partial \boldsymbol{A}}\boldsymbol{B}_{ti} + \boldsymbol{B}_{ri}\frac{\partial \boldsymbol{B}_{ti}}{\partial \boldsymbol{A}}\right)H(\phi_i) \qquad (5)$$

In the case of the triangular finite element, the radial and tangential component of magnetic flux is achieved as

$$\frac{\partial \boldsymbol{B}_r}{\partial \boldsymbol{A}_i} = \frac{1}{2\Delta}c_i \sin\theta - \frac{1}{2\Delta}b_i \cos\theta$$
$$\frac{\partial \boldsymbol{B}_t}{\partial \boldsymbol{A}_i} = -\frac{1}{2\Delta}c_i \cos\theta - \frac{1}{2\Delta}b_i \sin\theta \qquad (6)$$

And $\tilde{\lambda}$ may be obtained by solving

$$\boldsymbol{K}(\boldsymbol{\nu})\tilde{\lambda} = \frac{\partial T}{\partial \boldsymbol{A}} \qquad (7)$$

In the case of the triangular finite element, the elements of the stiffness matrix $\boldsymbol{K}$ are directly proportional to reluctivities.

$$\frac{\partial k_{lh}}{\partial \alpha} = \frac{1}{4\Delta}(b_l b_h + c_l c_h) \qquad (8)$$

The analysis results of the discrete design sensitivity based on adjoint variable method may be obtained by substituting (6) to (8) into (5).

The expressions of continuum design sensitivity analysis are shown as [5].

$$\frac{\partial \psi_{EM}}{\partial \mu} = \iiint_\Omega (g_\mu + \nabla\times\boldsymbol{A}\bullet\frac{1}{\mu^2}\nabla\times\boldsymbol{\lambda})d\Omega$$
$$\frac{\partial \psi_{EM}}{\partial \boldsymbol{J}} = \iiint_\Omega (g_{\boldsymbol{J}} + \boldsymbol{\lambda})d\Omega$$
$$\frac{\partial \psi_{EM}}{\partial \boldsymbol{M}} = \iiint_\Omega (g_{\boldsymbol{M}} + \nabla\times\boldsymbol{\lambda})d\Omega \qquad (9)$$

Where, μ, J, M is the design variable based on topology optimization, g_μ, g_J, g_M are the partial derivative of the optimization objective to the design variables, respectively. $\boldsymbol{\lambda}$ is the virtual magnetic vector potential, which has the same dimension with the original system. From (9), only passing through two steps of the finite element analysis by using continuum design sensitivity analysis, may the design sensitivity be acquired. The detailed derivation will be introduced in full paper.

V. OPTIMIZATION RESULT

The initial pole shape of the BLDC motor is shown in Fig. 2. The average magnetic density of the BLDC motor is used as constraint in order to avoid the sharp reduction of the magnetic density in the airgap. In this case, the average magnetic density should be greater than 0.3T. Fig. 2 is the optimized stator pole shape and magnetic flux with the level set method. In Fig. 3, the part with blue color represents ferromagnetic material, while the part with red color represents the airgap. The stator pole shape by using the level set avoids the result like the chessboard by using ON/OFF method and variable density method. Besides, the boundary shape after optimization is much smoother. Fig. 4 shows the cogging torques before and after topology optimization. It can be seen that the fluctuation of cogging torque is reduced after optimization. The detailed discussion will be presented in the full paper.

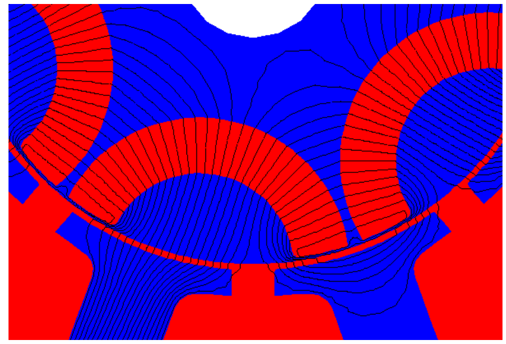

Fig.2. Initial pole shape of BLDC motor

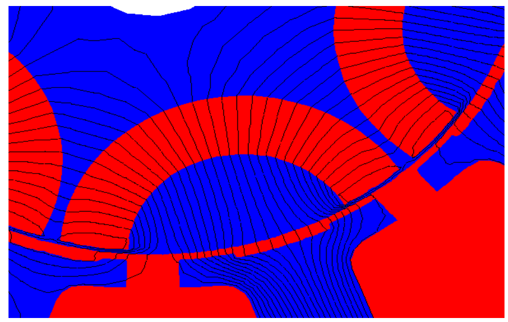

Fig.3. Optimal result of the stator pole shape and magnetic field distribution

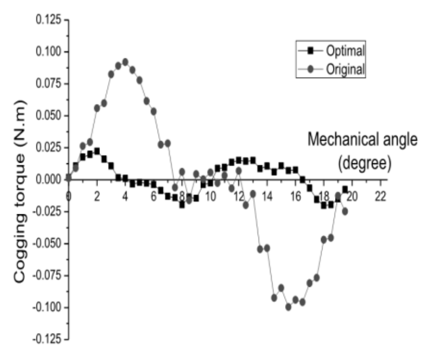

Fig.4. Cogging torque before and after topology optimization

VI. REFERENCES

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